

# Sixth Grade Math with Confidence

## Pilot Test, Unit 13B

*Note: There's no new memory work, so I haven't included the memory work list.*

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# Unit 13B: Rates

## Overview

Your child will learn how to calculate rates and solve problems involving rates. She'll explore a wide variety of real-world rates, including flow rate, typing rates, and heart rates. She'll also solve multi-step word problems involving unit prices and speed.

## What Your Child Will Learn

In this unit your child will learn to:

- Understand that a rate compares two quantities with different units
- Solve rate problems with multiplication and division and/or ratio tables
- Divide to find unit rates
- Manipulate units correctly when solving rate problems
- Solve problems involving speed, time, and distance
- Solve multi-step word problems involving unit prices or speed

## Lesson List

*Lesson 13.1	Introduce Rates
*Lesson 13.2	Use Rates to Solve Problems
*Lesson 13.3	Unit Rates
*Lesson 13.4	Unit Prices
Lesson 13.5	Speed, Time, and Distance
Lesson 13.6	Speed Word Problems
Lesson 13.7	Enrichment (Optional)

*\*Included in Unit 13A*

## Extra Materials Needed for Unit 13B

- For optional Enrichment Lesson:
  - *What's the Point of Math?* DK Publishing, 2020.
  - Distance tracking app, map, or map app, optional
  - Timer or timer app

## Lesson 13.5 – Speed, Time, and Distance

PURPOSE	MATERIALS
<ul style="list-style-type: none"> <li>Explore the relationship between speed, time, and distance</li> <li>Multiply or divide to solve problems that involve speed, time, and distance</li> </ul>	<ul style="list-style-type: none"> <li>None</li> </ul>
<ul style="list-style-type: none"> <li>What do we call a letter that stands for a number? <i>A variable.</i></li> <li>What do we call a number multiplied by a variable? <i>A coefficient.</i></li> <li>What do we call the parts of an expression that are added or subtracted? <i>Terms.</i></li> <li>What do we call a term with only a number and no variable part? <i>A constant.</i></li> </ul>	

### Warm-up (A): Explore Speed with a Bug Race

This warm-up activity helps your child kinesthetically understand the difference between different speeds. Make sure not to skip this simple hands-on activity.

In the last two lessons, you learned how to find unit rates and unit prices. You wrote each quantity above or below the fraction bar and then divided to find the unit rate.

Today, you'll learn how to solve problems that involve speed, time, and distance. We'll warm up today by finding bug's speeds in a 6-inch bug race.

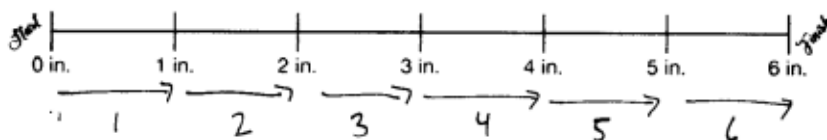
The ladybug travels the 6 inches in 6 seconds. We want to find the ladybug's speed in inches per second. So, write the number of inches above the fraction bar and the number of seconds below the fraction bar. Have your child find the ladybug's speed and complete the blank.

The ladybug travels  
6 inches in 6 seconds.

$$\frac{6 \text{ in.}}{6 \text{ sec.}} = 1 \frac{\text{in.}}{\text{sec.}}$$

1 inches per second

What is the ladybug's speed? *1 inch per second.* I'll use my finger to act out the ladybug's race. Put your finger at the Start position. Say "Start" and then gradually move your finger along the race course at a speed of 1 inch per second. Count "1 Mississippi" as you move from 0 to 1, "2 Mississippi" as you move from 1 to 2, and so on. Then, have your child act out the ladybug's race in the same way.



Caption: Count each number as you move your finger the corresponding distance.

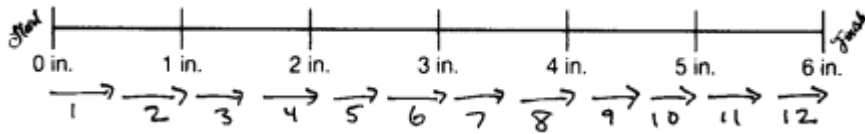
Feel free to use whatever word or phrase your family usually uses to count seconds accurately (like “potato” or “one thousand”).

Have your child calculate the snail’s speed and trace the snail’s race with her finger. Then, repeat for the ant.

The snail travels 6 inches in 12 seconds.

$$\frac{6 \text{ in.}}{12 \text{ sec.}} = \frac{1}{2} \frac{\text{in.}}{\text{sec.}}$$

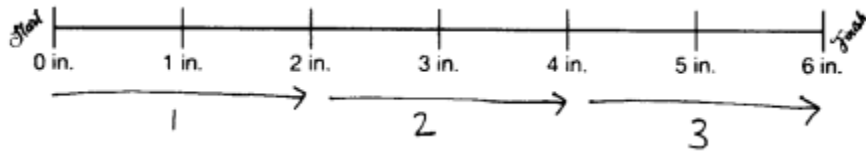
$\frac{1}{2}$  inches per second



The ant travels 6 inches in 3 seconds.

$$\frac{6 \text{ in.}}{3 \text{ sec.}} = 2 \frac{\text{in.}}{\text{sec.}}$$

2 inches per second



Which animal is fastest? *The ant.* Which animal is slowest? *The snail.* Which bug won the race? *The ant.*

### Activity (B): Find Speed, Time, and Distance

Speed is a special type of rate that compares a distance to one unit of time. We solve speed problems with multiplication and division, just like the rate problems you solved in Lesson 13.2.

Read aloud the first example problem. In this problem, we know the plane’s speed and how long the plane traveled. What do we want to find? *The distance the plane traveled.*

**Ex.** The airplane travels at a speed of 600 miles per hour. If the plane flies for  $1\frac{1}{2}$  hours, how many miles does the plane travel?

miles	600	?
hours	1	$1\frac{1}{2}$

The double number line shows the number of miles and the number of hours. The first tick mark shows that the plane travels 600 miles in 1 hour. The second tick mark shows that we want to find how far the plane flies in 1 and one-half hours.

We can use multiplication to solve the problem, just like we solve other rate problems.

- The speed tells us how far the plane travels in 1 hour, so we multiply the speed by the number of hours. 600 times 1 and one-half is 900.

- We also multiply the units. When we multiply miles per hour times hours, we have hours above and below the fraction bar. The hours cancel, and so we get the correct unit: miles.
- How far did the plane fly? *900 miles*.
- Point to the cloud below the example. When we know the speed and time, we multiply the speed by the time to find the distance.

$600 \frac{\text{miles}}{\text{hour}} \times 1 \frac{1}{2} \text{ hours}$   
 $600 \frac{\text{miles}}{\text{hour}} \times \frac{3}{2} \text{ hours} = 900 \text{ miles}$   
 speed  $\times$  time = distance

If your child has trouble seeing how the units cancel, write out the calculation on a piece of scrap paper as shown.

$$600 \frac{\text{miles}}{\text{hour}} \times 1 \frac{1}{2} \text{ hours} = \frac{600 \text{ miles}}{1 \text{ hour}} \times \frac{1 \frac{1}{2} \text{ hours}}{1} = \frac{600 \text{ miles} \times 1 \frac{1}{2} \text{ hours}}{1 \text{ hour}} = 900 \text{ miles}$$

Read aloud the second example problem. In this problem, we know the plane's speed and how far the plane travels. What do we want to find? *How long the plane travels to go this distance.* How is this problem different than the first problem? *Sample answer: In the first problem, we wanted to find how far the plane travels. In this problem, we want to find out how long it takes the plane to go a certain distance.*

**Ex.** The airplane travels at a speed of 600 miles per hour. How long does it take the plane to travel 2,400 miles?

The first tick mark in the double number line again shows that the plane travels 600 miles in 1 hour. The second tick mark shows that we want to find how long it takes the plane to travel 2,400 miles.

We use division to solve this problem, just like other rate problems.

- We know the plane travels 600 miles in 1 hour. So, we divide 2,400 by 600 to find how many hours it takes the plane to travel 2,400 miles.
- 2,400 divided by 600 equals 4.
- Miles divided by miles per hour equals hours.
- How long does it take the plane to travel 2,400 miles? *4 hours*.
- Point to the cloud below the example. When we know the distance and speed, we divide the distance by the speed to find the time.

$$2,400 \text{ miles} \div 600 \frac{\text{miles}}{\text{hour}} = 4 \text{ hours}$$

distance  $\div$  speed = time

If your child is curious about why miles divided by miles per hour equals hours, demonstrate how to divide the units.

$$2,400 \text{ miles} \div 600 \frac{\text{miles}}{\text{hours}}$$

$$2,400 \text{ miles} \div \frac{600 \text{ miles}}{1 \text{ hour}}$$

$$2,400 \text{ miles} \times \frac{1 \text{ hour}}{600 \text{ miles}} = \frac{2,400 \cancel{\text{miles}} \times 1 \text{ hour}}{600 \cancel{\text{miles}}} = 4 \text{ hours}$$

It's usually simplest to use multiplication and division to solve speed problems, so these examples do not demonstrate how to use a ratio table. However, if your child prefers using ratio tables to solve rate problems, it's fine for her to use a ratio table to set up the problems before multiplying or dividing.

### Activity (C): Complete a Running Log

Speed problems always involve three quantities: speed, time, and distance. If you know two of them, you can find the third.

This runner only wrote down two of the three quantities for each of her workouts. We'll use the two quantities we know to find the quantity we don't know.

Have your child use multiplication and division to complete the table. Have her write her equations in the work space.

Running Log	Speed (miles per hour)	Time (hours)	Distance (miles)
Monday	6	$2\frac{1}{2}$	3
Tuesday	6	3	18
Thursday	$5\frac{1}{2}$	4	22
Saturday	8	$1\frac{1}{4}$	10

$6 \times 2\frac{1}{2}$   
 $3\cancel{6} \times \frac{5}{\cancel{2}} = 3 \text{ mi.}$   
 $18 \text{ mi.} \div 6 \frac{\text{mi.}}{\text{hr.}} = 3 \text{ hr.}$   
 $\frac{22 \text{ mi.}}{4 \text{ hr.}} = 5\frac{2}{4} = 5\frac{1}{2} \frac{\text{mi.}}{\text{hr.}}$   
 $10 \text{ mi.} \div 8 \frac{\text{mi.}}{\text{hr.}} = \frac{10}{8} = 1\frac{1}{4} \text{ hr.}$

Lesson 13.5

Your child will memorize the formula that relates speed, time, and distance in Lesson 13.6. For now, simply encourage her to use what she knows about rates to solve the problems. She may also refer back to the completed problems in parts A and B.

For example, for Monday: **She ran at a speed of 6 miles per hour for 2 and one-half hours. What do we want to find? *The distance.* We know how fast she ran and for how long, so we multiply the speed by the time to find the distance.**

### **Independent Practice and Review**

Have your child complete the Lesson 13.5 Practice and Review workbook pages.

## Lesson 13.6 – Speed Word Problems

PURPOSE	MATERIALS
<ul style="list-style-type: none"> <li>• Memorize the basic formula for speed, time, and distance</li> <li>• Solve multi-step problems that involve speed, time, and distance</li> </ul>	<ul style="list-style-type: none"> <li>• None</li> </ul>
<ul style="list-style-type: none"> <li>• What decimal is equivalent to one-half? <math>0.5</math>.</li> <li>• What decimal is equivalent to one-fourth? <math>0.25</math>.</li> <li>• What decimal is equivalent to three-fourths? <math>0.75</math>.</li> <li>• What decimal is equivalent to one-fifth? <math>0.2</math>.</li> <li>• What decimal is equivalent to two-fifths? <math>0.4</math>.</li> <li>• What decimal is equivalent to three-fifths? <math>0.6</math>.</li> <li>• What decimal is equivalent to four-fifths? <math>0.8</math>.</li> </ul>	

### Warm-up (A): Memorize the Basic Formula for Speed, Time, and Distance

In the last lesson, you solved problems that involve speed, time, and distance. Speed problems are common in everyday life, so it's helpful to memorize the basic formula that relates speed, time, and distance: speed times time equals distance.

When you were younger, you learned how to use multiplication problems to solve division problems. For example, if you know 8 times 7 equals 56, what's 56 divided by 7? 8. What's 56 divided by 8? 7.

$$8 \times 7 = 56 \quad \begin{array}{l} \rightarrow 56 \div 7 = 8 \\ \rightarrow 56 \div 8 = 7 \end{array}$$

You can use the same kind of thinking to use the basic formula for speed, time, and distance to figure out other versions of the formula. Show your child part A. Speed times time equals distance. So, what does distance divided by time equal? *Speed*. What does distance divided by speed equal? *Time*.

$$\begin{array}{l} \text{speed} \times \text{time} = \text{distance} \quad \begin{array}{l} \rightarrow \text{distance} \div \text{time} = \underline{\text{Speed}} \\ \rightarrow \text{distance} \div \text{speed} = \underline{\text{Time}} \end{array} \end{array}$$

Cover part A with your hand. Have your child try to answer the following questions without looking at part A. Allow him to peek if needed.

- What's the basic formula for speed? *Speed times time equals distance.*
- What does distance divided by time equal? *Speed.*
- What does distance divided by speed equal? *Time.*

We'll practice these formulas in the memory work so that you learn them by heart. But, if you ever forget them, you can also use what you know about rates to figure out speed problems.

## Activity (B): Solve Multi-Step Speed Word Problems

Today, you'll learn how to solve speed word problems. Read aloud the first example problem.

**Ex.** Thomas runs for  $\frac{1}{2}$  of an hour at a speed of 6 miles per hour. Then, he walks the same distance home at a speed of 3 miles per hour. How long does it take him to walk home?

There is a lot going on in this problem! It's important to make sure you understand the problem and identify the goal before you start solving.

- What happens first in this problem? *Thomas runs for half of an hour at 6 miles per hour.*
- What happens second? *He walks home at 3 miles per hour.*
- This diagram organizes the numbers in the problem. It shows that he traveled the same distance in both directions. First, he went one way at a speed of 6 miles per hour. It took him one-half of an hour. Then, he came back the same distance at a speed of 3 miles per hour.
- What is the goal of the problem? *Find how long it takes him to walk home.*



This problem has two steps.

- First, we find the distance he runs. He ran at a speed of 6 miles per hour for one-half of an hour. 6 miles per hour times one-half of a mile equals 3 miles. He ran 3 miles.
- Then, we use that distance and his walking speed to find how long it takes him to walk back home. 3 miles divided by 3 miles per hour equals 1 hour.

$$6 \frac{\text{miles}}{\text{hour}} \times \frac{1}{2} \text{ hour} = 3 \text{ miles}$$

$$3 \text{ miles} \div 3 \frac{\text{miles}}{\text{hour}} = 1 \text{ hour}$$

Remember, speed problems always have three parts: speed, time, and distance. We needed to find the distance first so that we could use the speed and distance to find the time.

Read aloud the second example question. **What is the goal of this problem?** *Find his average speed for the whole trip.*

**Ex.** What is Thomas' average speed for the whole trip?

We divide distance by time to find speed.

- First, we find the total distance and total time.
- He traveled 3 miles in either direction, so how far did he travel in all? *6 miles.*
- He ran for one-half of an hour and walked for 1 hour, so how much time did he spend on the trip in all? *1 and one-half hours.*
- Next, we divide the total distance by the total time to find the average speed for the whole trip.
- 6 divided by 1 and one-half equals 4. Miles divided by hours equal miles per hour. So, what's his average speed for the whole trip? *4 miles per hour.*

Total distance:  $3 \text{ mi.} + 3 \text{ mi.} = 6 \text{ mi.}$

Total time:  $\frac{1}{2} \text{ hr.} + 1 \text{ hr.} = 1 \frac{1}{2} \text{ hr.}$

Average speed:  $\frac{6 \text{ miles}}{1 \frac{1}{2} \text{ hours}}$

$6 \div 1 \frac{1}{2} \rightarrow 6 \div \frac{3}{2}$

$6 \times \frac{2}{3} = \frac{12}{3} = 4 \frac{\text{miles}}{\text{hour}}$

In this case, we had to find the total distance and time before we could divide to find the speed.

This may be the first time your child has seen a mixed number below the fraction bar. If this confuses your child, remind him that we interpret the fraction bar as meaning division in rate problems (and not as describing a relationship between a part and a whole). **6 miles over 1 and one-half hours is just another way to write 6 miles divided by 1 and one-half hours.**

Have your child read the first part of the practice problem aloud.

- **What happens first in this problem?** *Marianne rides for 1 hour at 30 kilometers per hour.*
- **What happens second?** *She bikes home the same distance.*
- **What is the goal of the problem?** *Find how far she bikes in all.*

**Let's make a simple diagram to organize the information.** Write 30 km/hr. and 1 hr. above the line as shown. Draw a short arrow to show the direction of travel.

**We don't know anything about her return trip yet, so I'll just draw an arrow in the other direction to remind us that she biked back, too.** Draw a short arrow in the other direction as shown.

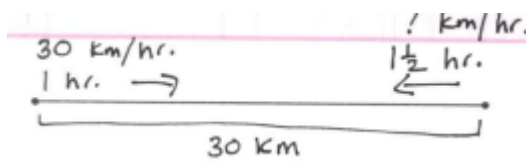
Marianne rode her bike at a speed of 30 kilometers per hour for 1 hour. Then, she biked the same distance home. How far did Marianne bike in all?



How can we find how far she biked in all? *Sample answer: Find how far she biked in one direction and then double it. How can we find how far she biked in one direction? Multiply speed times time.* Have your child multiply  $30 \text{ km/hr.} \times 1 \text{ hr.}$  to find the distance in one direction. Then, have him double it to find the total distance. **How far did Marianne bike in all? 60 kilometers.** Draw a bracket below the diagram and label it “30 km.”

Marianne rode her bike at a speed of 30 kilometers per hour for 1 hour. Then, she biked the same distance home. How far did Marianne bike in all?

60 km



$$30 \frac{\text{km}}{\text{hr.}} \times 1 \text{ hr.} = 30 \text{ km}$$

Have your child read aloud the next part of the problem.

- **What do we learn in this part of the problem?** *It takes Marianne 1 and one-half hours to bike home.*
- **What is the goal of this part of the problem?** *Find her speed for the return trip.*
- **Let’s label the diagram to match.** Write “? km/hr.” and “1 1/2 hr.” above the line as shown.
- **Do you expect her speed for the return trip will be greater than or less than 30 kilometers per hour? Why?** *Sample answer: It will be less than 30 kilometers per hour, because the second part of the trip takes longer than the first part.*

**How can we find how fast Marianne biked home?** *Divide the distance by the time.* Have your child divide 30 kilometers by 1 1/2 hours to find her speed for the return trip. Remind her as needed to use 30 kilometers (and not 60 kilometers) for the distance, because 30 kilometers is the length of one portion of the trip.

It took Marianne <sup>1 1/2</sup> hours to bike home. What was her average speed for the return trip?

20 km/hr.

$$\frac{30 \text{ km}}{1 \frac{1}{2} \text{ hr.}} \rightarrow 30 \div 1 \frac{1}{2}$$

$$30 \div \frac{3}{2}$$

$$30 \times \frac{2}{3} = 20 \text{ km/hr.}$$

Have your child read aloud the final part of the problem. **What is the goal of this part of the problem?** *Find her average speed for the whole trip.* Have your child find the total distance and total time. Then, have him divide the total distance by the total time to find Marianne’s average speed for the entire trip. **What is Marianne’s average speed for the whole trip? 24 kilometers per hour.**

What was Marianne's average speed for the whole trip?

24 km/hr.

$$\frac{60 \text{ km}}{2\frac{1}{2} \text{ hr.}} \quad 60 \div 2\frac{1}{2}$$

$$60 \div \frac{5}{2}$$

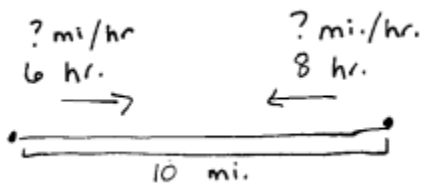
$$120 \times \frac{2}{5} = 24 \text{ km/hr.}$$

Lesson 13.6

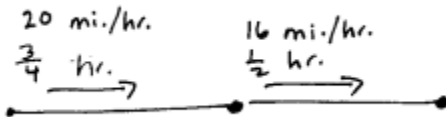
## Independent Practice and Review

Have your child complete the Lesson 13.6 Practice and Review workbook pages.

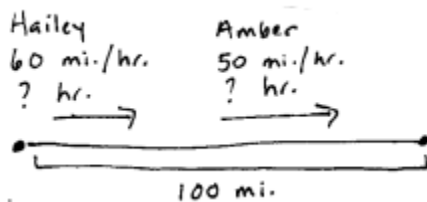
Your child is not required to draw diagrams for the word problems on the Practice page, but he may if he finds it helpful.



Caption: Sample diagram for the Lauren problem.



Caption: Sample diagram for the Jacob problem.



Caption: Sample diagram for the Hailey and Amber problem.

## Lesson 13.7 – Enrichment (Optional)

PURPOSE	MATERIALS
<ul style="list-style-type: none"><li>• Review memory work</li><li>• Appreciate how math can be used in real-life situations</li><li>• Enrichment</li><li>• Summarize what your child has learned and assess your child’s progress</li></ul>	<ul style="list-style-type: none"><li>• Distance tracking app, map, or map app, optional</li><li>• Timer or timer app</li><li>• <i>What’s the Point of Math?</i></li></ul>

### Warm-up: Review Memory Work

Quiz your child on any of the memory work items that she struggled with during this unit.

### Math Book: *What’s the Point of Math?*

Read pages 108-111 in *What’s the Point of Math?*

### Enrichment Activity: Find Your Walking, Running, or Biking Speed

In this unit, you’ve calculated a lot of speeds. Today, you’ll find your own speed!

Have your child decide whether she’d like to walk, run, or bike for this activity. Then, have her choose where and how far she’d like to run, walk, or bike (based on her fitness level and the options are available to you).

Decide how you will measure the distance she travels.

- If you have a fitness tracker app, have your child carry the device so that it can measure the distance during the trip.
- If you have a map app, plot the trip in the app and have the app calculate the distance.
- If you prefer not to use technology for this activity, use a paper map to approximate the distance. Or, have your child travel a distance that is already measured (such as a marked hiking trail or a track at an athletic facility).

Use a timer or timer app to measure how long it takes your child to complete the activity. Then, have her divide the distance by the time to find her speed. For example: *I walked 2 miles in 50 minutes. So, I traveled one-twenty-fifth of a mile per minute.*

$$\frac{2 \text{ miles}}{50 \text{ minutes}} = \frac{1}{25} \text{ miles/minute}$$

If your child would like to express her speed in miles per hour, show her how to multiply her speed in miles per minute by 60. **60 minutes equal 1 hour. Multiplying by 60 minutes over 1 hour converts the speed to miles per hour. The minutes cancel, and we’re left with miles per hour.**

$$\frac{1 \text{ mile}}{25 \text{ minutes}} \times \frac{60 \text{ minutes}}{1 \text{ hour}} = \frac{60 \text{ miles}}{25 \text{ hours}} = 2 \frac{2}{5} \text{ miles/hour}$$

### Unit Wrap-Up

Have your child complete the Unit 13 Wrap-Up.

# Unit 13 Checkpoint

## What to Expect at the End of Unit 13

By the end of Unit 13, most children will be able to do the following:

- Understand that a rate compares two quantities with different units.
- Solve rate problems with multiplication and division and/or ratio tables. (Either way is fine.)
- Divide to find unit rates. Many children will occasionally write the numbers in the wrong places above and below the fraction bar.
- Write the answers to rate problems with the correct units. Many children will still rely on logical thinking rather than understanding how to cancel or manipulate the units.
- Solve problems involving speed, time, and distance.
- Solve multi-step word problems involving unit prices or speed. Many children will still need coaching when solving problems with more than one step.

## Is Your Child Ready to Move on?

In Unit 14, your child will learn how to multiply and divide decimals by decimals. Your child will continue to practice rate and speed problems in the Review pages.

