Sixth Grade Math with Confidence
Pilot Test, Unit 15B

Pilot test note: We're almost there! Unit 15C will have the last three lessons in the book.

Unit 15: Data and Statistics

Overview

Your child will learn how to use statistics to describe a data set. First, she'll learn how to use mean, median, and mode to describe the center of the data set. Then, she'll learn how to use mean absolute deviation, quartiles, and interquartile range to describe how spread out the data set is. She will also learn several different ways to represent and visualize data, including dot plots, histograms, and box plots.

In the final lessons, you and your child will review the major concepts and skills she has learned this year and celebrate completing the book.

What Your Child Will Learn

In this unit your child will learn to:

- Find the mean, median, and mode for a data set and understand that these statistics describe the center of the data set
- Find the range, mean absolute deviation, quartiles, and interquartile range for a data set and understand that these statistics describe the spread of the data set
- Create and interpret dot plots, histograms, and box plots
- Describe the shape of a data set as right-skewed, left-skewed, or symmetric and use words like gap, peak, and outlier to describe its features
- Use statistics to analyze data, compare two data sets, and draw conclusions

Lesson List

Lesson 15.1*	Mean, Median, and Mode
Lesson 15.2*	Describe Distributions
Lesson 15.3*	Histograms
Lesson 15.4*	Mean Absolute Deviation
Lesson 15.5	Quartiles
Lesson 15.6	Interquartile Range
Lesson 15.7	Box Plots
Lesson 15.8	Enrichment (Optional)
Lesson 15.9**	Review Fractions, Decimals, and Percentages
Lesson 15.10**	Review Ratios and Rates
Lesson 15.11**	Review Geometry, the Coordinate Plane, Variables, and Data

**Included in Unit 15A.* ***Coming in Unit 15C.*

The optional enrichment lesson for this unit is in the middle of the unit so you can wrap up your study of data before beginning the end-of-year review lessons. This enrichment lesson involves gathering data and using statistics to summarize the results. If you'd like to complete Lesson 15.8 in one day, have your child conduct the survey *before* you teach Lesson 15.8. See Lesson 15.8 (page X) for directions on how to help your child choose a question and gather the data.

Lesson 15.5 – Quartiles

PURPOSE	MATERIALS
• Split data into 4 equal groups to preview quartiles • Understand that quartiles are numbers that split a data set as equally as possible into four equal groups • Find the quartiles for a data set	• 9 slips of paper • 3 writing utensils or craft sticks
• **What does mean absolute deviation measure?** *How much the data in a data set varies from the mean.*	

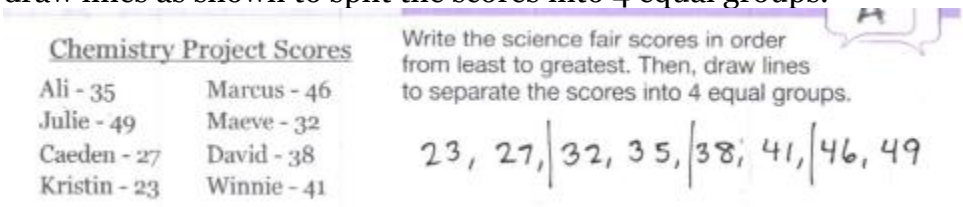
Lessons 15.5, 15.6, and 15.7 teach your child how to use quartiles to describe and visualize a data set's spread. Quartiles are the numbers that split a data set into four roughly-equal groups. By looking at how close together the quartiles are, we get a better sense of how spread out the values in the data set are.

You will use 9 slips of paper to demonstrate how to find quartiles in this lesson. If you'd like to prepare the slips of paper ahead of time, write the following numbers on separate slips of paper: 19, 28, 30, 36, 38, 40, 42, 45, 55.

Warm-up (A): Sort Data into Four Equal Groups

Show your child the chemistry project scores in part A. **At this science fair, the judges split the students into four equal groups. Each group gets a different level of award.**

Have your child write the scores in order. **How many scores are there?** *8*. **If we split the scores into 4 equal groups, how many scores will be in each group?** *2*. Have your child draw lines as shown to split the scores into 4 equal groups.



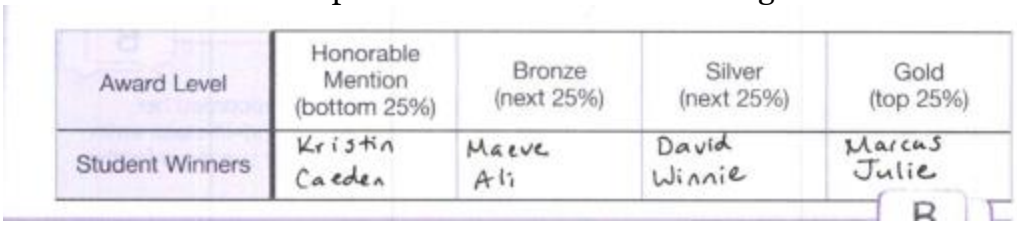
Chemistry Project Scores

Ali - 35	Marcus - 46
Julie - 49	Maeve - 32
Caeden - 27	David - 38
Kristin - 23	Winnie - 41

Write the science fair scores in order from least to greatest. Then, draw lines to separate the scores into 4 equal groups.

23, 27, | 32, 35, | 38, 41, | 46, 49

Students in the bottom 25% of the scores earn an honorable mention. Which students earn an honorable mention? *Kristin and Caeden*. **How do you know?** *Sample answer: 25% of 8 scores is 2. Kristin and Caeden have the lowest two scores*. Write Kristin and Caeden in the chart. Repeat with the other award categories.

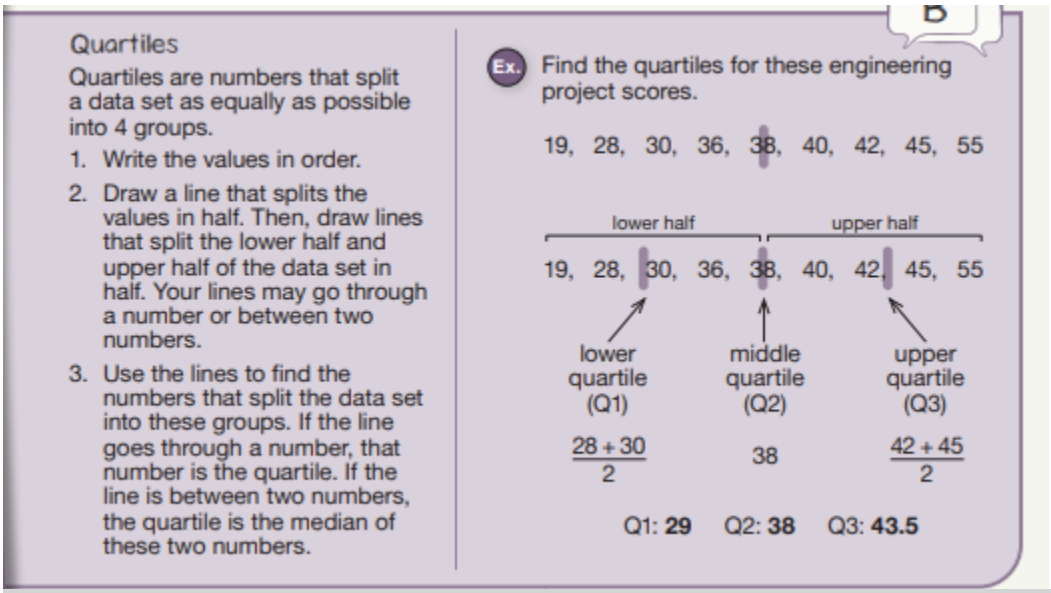


Award Level	Honorable Mention (bottom 25%)	Bronze (next 25%)	Silver (next 25%)	Gold (top 25%)
Student Winners	Kristin Caeden	Maeve Ali	David Winnie	Marcus Julie

Activity (B): Use Slips of Paper to Find Quartiles

Today, you'll learn how to find the quartiles for a data set. The word quartile comes from the Latin word *quartus*, which means fourth. It has the same root word as quarter and quart.

Quartiles are the numbers that split a data set into 4 groups. To find the quartiles, we divide the data set into 4 groups that are as equal as possible. Have your child read the text box aloud.



Quartiles

Quartiles are numbers that split a data set as equally as possible into 4 groups.

1. Write the values in order.
2. Draw a line that splits the values in half. Then, draw lines that split the lower half and upper half of the data set in half. Your lines may go through a number or between two numbers.
3. Use the lines to find the numbers that split the data set into these groups. If the line goes through a number, that number is the quartile. If the line is between two numbers, the quartile is the median of these two numbers.

Ex. Find the quartiles for these engineering project scores.

19, 28, 30, 36, 38, 40, 42, 45, 55

lower half | upper half

19, 28, 30, 36, 38, 40, 42, 45, 55

lower quartile (Q1): $\frac{28+30}{2}$

middle quartile (Q2): 38

upper quartile (Q3): $\frac{42+45}{2}$

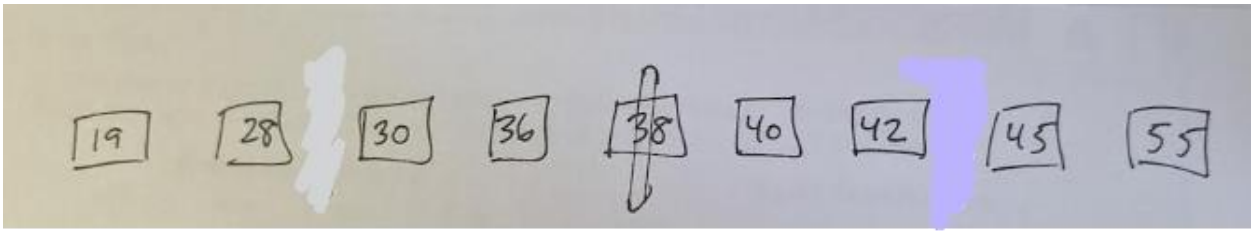
Q1: **29** Q2: **38** Q3: **43.5**

The word quartile is sometimes used to describe each of the 4 equal groups of values, rather than the numbers that divide the values into 4 groups. In *Math with Confidence*, quartile always refers to the numbers that split the data set into 4 equal (or roughly-equal) groups.

The directions for finding quartiles look complicated, but it's actually a pretty simple process. We'll use slips of paper to model the example problem. First, we'll divide the data set as evenly as possible into 4 groups. Then, we'll find the quartiles.

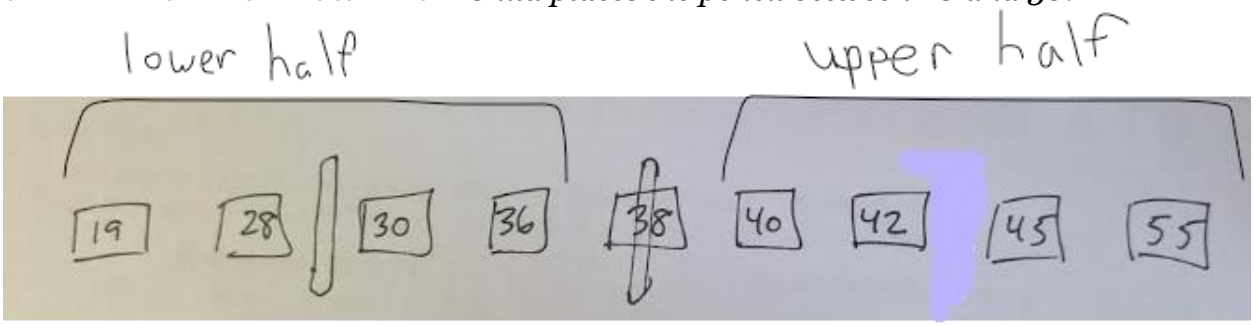
Write the engineering project scores from the example on separate slips of paper: 19, 28, 30, 36, 38, 40, 42, 45, 55. Place the slips of paper in order on the table.

How many scores are there? *9.* **9 is odd, so the median will be one of the scores. Which score is the median?** *38.* Place a pencil on 38. **There are 4 values on either side of 38, so placing the pencil on 38 splits the data set in half as evenly as possible.**

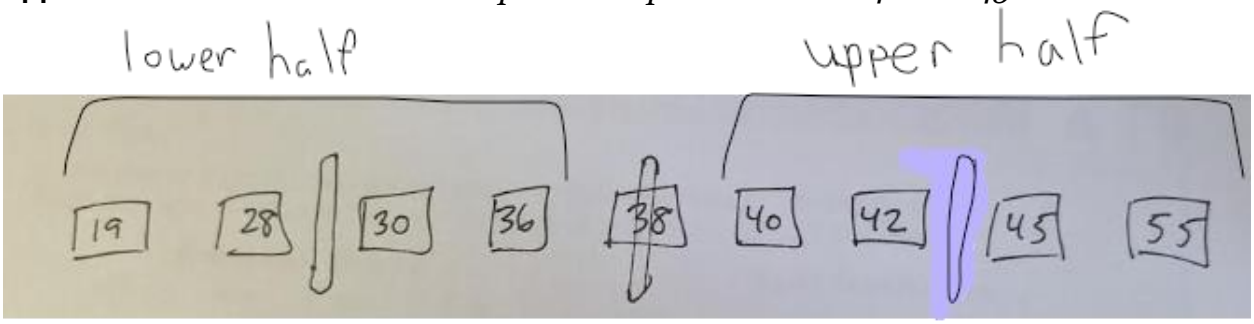


Next, we split the lower half of the data set in half. The lower half of the data set is all the values below the median. Which values are in the lower half of the data set? *19, 28,*

30, and 36. Give your child another pencil or craft stick. **Place the pencil so that it splits the lower half of the data in half.** *Child places the pencil between 28 and 30.*



Next, we split the upper half of the data set in half. The upper half of the data set is all the values above the median. Which values are in the upper half of the data set? *40, 42, 45, 55*. Give your child another pencil or craft stick. **Place the pencil so that it splits the upper half of the data in half.** *Child places the pencil between 42 and 45.*

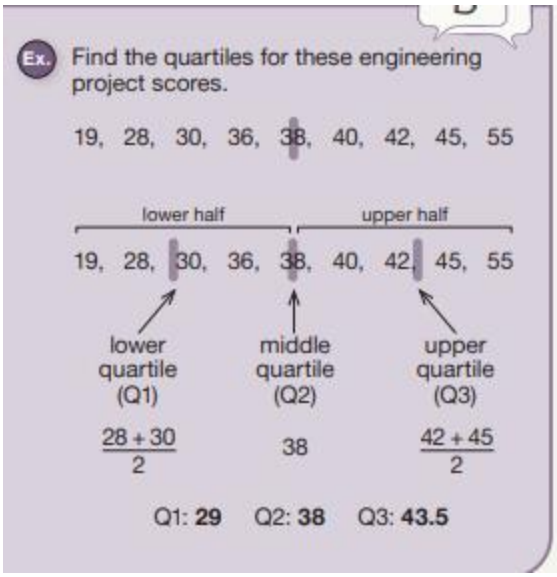


Now, we use the dividing lines to find the quartiles. If the line goes through a number, that number is the quartile. If the line is between two numbers, the quartile is the median of those two numbers.

- **The first pencil is between 28 and 30. The median of 28 and 30 is 29, so the lower quartile is 29.**
- **The second pencil is on 38, so the middle quartile is 38.**
- **The third pencil is between 42 and 45. The median of 42 and 45 is 43.5, so the upper quartile is 43.5.**

You can also think of this process as finding three medians: the median of the *whole* data set, the median of the *lower half* of the data set, and the median of the *upper half* of the data set.

Show your child the printed example. **We often number the quartiles. The lower quartile is called Q1, the middle quartile is called Q2, and the upper quartile is called Q3.**



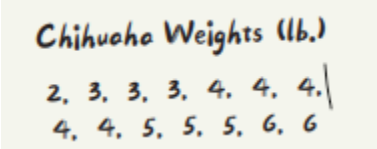
Which quartile is also the median? *The middle quartile. Q2.* **The median of a data set splits the data set in half, so the middle quartile is also the median.**

Activity (B): Find Quartiles for Dogs' Weights

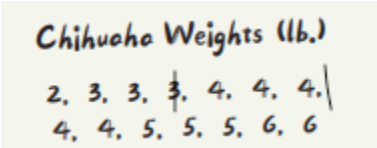
Now, we'll find the quartiles for two data sets about dogs' weights. One data set has chihuahua's weights, and the other data set has golden retriever's weights.

Use the following questions to help your child find the quartiles for the chihuahuas' weights:

- **How many values are there in the data set for chihuahuas?** *14.* **Is 14 divisible by 4?** *No.* **We can't divide 14 values evenly into 4 groups, but we'll make the groups as even as possible.**
- **14 is an even number, so your first dividing line will be between 2 values.** Have your child draw a line between the seventh and eighth values.

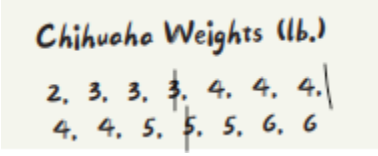


- **Now, we divide the lower half of the values in half. How many values are in the lower half?** *7.* **7 is an odd number, so your dividing line will be on one of the values.** Have your child draw a line through the fourth value (in the lower half) as shown.

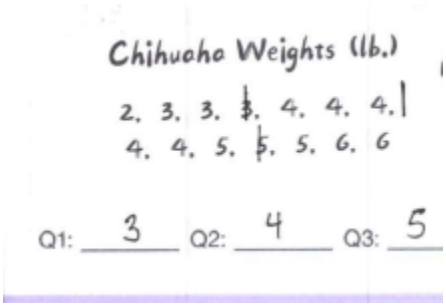


- **Then, we divide the upper half of the values in half. How many values are in the upper half?** *7.* **7 is an odd number, so your dividing line will be on one of the values**

again. Have your child draw a line through the fourth value (in the upper half) as shown.

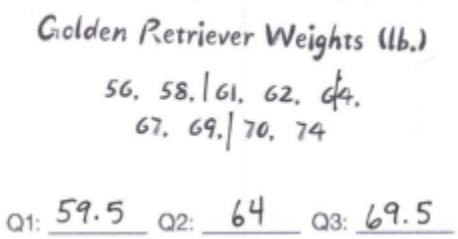


- **Last, we use the lines to find the quartiles. The first line goes through 3, so Q1 is 3.**
- **The second line is between 4 and 4. If the line is between two numbers, the quartile is the median of those two numbers. What's the median of 4 and 4?** *4.*
- **The third line goes through 5, so Q3 is 5.**



If your child has trouble splitting the printed numbers into groups, write the numbers on slips of paper instead.

How many values are there in the data set for the golden retrievers? *9.* **9 is an odd number, so your first dividing line will go through one of the values.** Repeat with the golden retriever weights.



Which type of dog is heavier, chihuahuas or golden retrievers? *Golden retrievers.*
Which type of dog's weights are more spread out? *Golden retrievers.* **In the next lesson, you'll learn how to use quartiles to measure how spread out data sets are.**

Independent Practice and Review

Have your child complete the Lesson 15.5 Practice and Review workbook pages.

Lesson 15.6 – Interquartile Range

PURPOSE	MATERIALS
• Practice finding quartiles • Find the interquartile range and five-number summary for a data set	• Playing cards

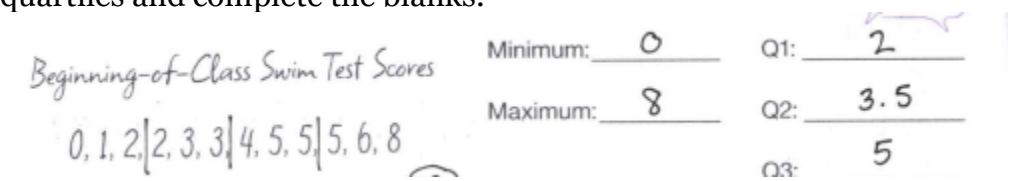
- **What is the mean of a data set?** *The sum of the values divided by the number of values.*
- **What is the median of a data set?** *The middle value when the values are in order from least to greatest.*
- **What is the mode of a data set?** *The value that occurs most frequently.*

Interquartile range is a way to use quartiles to describe how spread out the middle 50% of a data set is. If you're unfamiliar with the term, make sure to read the scripted lesson word-for-word to teach the concept to your child.

Warm-up (A): Practice Finding Quartiles

At swim lessons, students often need to pass a skills test before they move to the next level. This data set shows how the students in a swim class performed on a test at the beginning of the session. In part B, we'll see how the students performed on the same test at the end of the session.

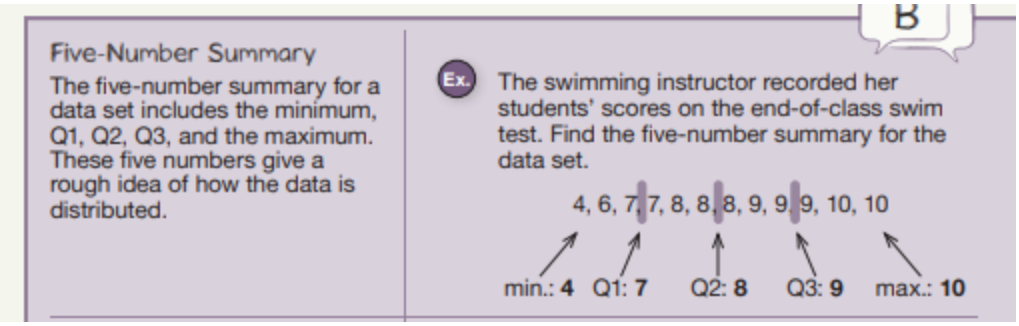
How many values are there in this test score data set? *12*. Have your child draw lines as shown to split the values into four groups. **All of the lines are between values, so you find the median of the numbers on each side of each line.** Then, help him find the quartiles and complete the blanks.



Activity (B): Find the Five-Number Summary and Interquartile Range for a Data Set

In the last lesson, you learned how to find quartiles. Today, you'll learn how to find the five-number summary and interquartile range for a data set.

The five-number summary for a data set includes the minimum, the 3 quartiles, and the maximum. These five numbers give us a summary of how spread out the values are. Have your child read the top of the text box aloud.



The reason we use these five numbers is because they split the data into intervals that each contain 25% of the values. These five numbers are the endpoints of the intervals.

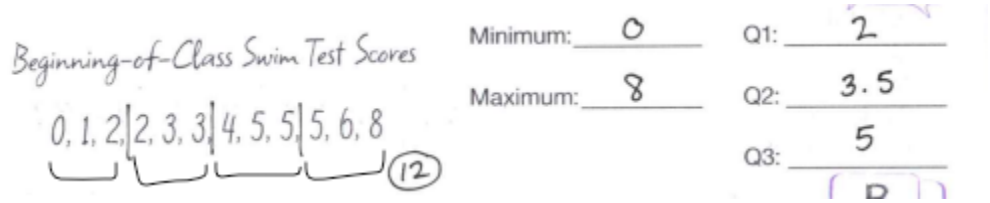
- **What are the five numbers in the five-number summary for this data set?** *4, 7, 8, 9, 10.*
- **The bottom 25% of the scores are between 4 and 7.**
- **The next 25% of scores are between 7 and 8.**
- **The next 25% of scores are between 8 and 9.**
- **The upper 25% of scores are between 9 and 10.**

Caption: Each bracket contains 25% of the values.

In English, the word "between" can include or exclude the endpoints of an interval. For example, if you say, "All the children are between 5 and 13 years old," you might mean that the youngest child is 5 and the oldest child is 13. Or, you might mean that all the children are older than 5 and younger than 13. In this unit, the lesson script uses the inclusive version of "between," where the endpoint numbers are included in the interval.

Point to part A. **In part A, you found the five-number summary for the beginning-of-class test scores.**

- **The bottom 25% of the scores are between which two numbers?** *0 and 2.*
- **The next 25% of scores are between which two numbers?** *2 and 3.*
- **The next 25% of scores are between which two numbers?** *4 and 5.*
- **The upper 25% of scores are between which two numbers?** *5 and 8.*

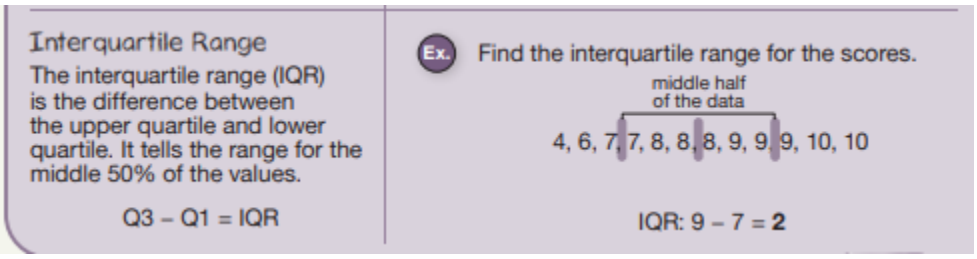


Caption: Each bracket contains 25% of the values.

The bottom part of the text box tells how to find the interquartile range for a data set. The prefix *inter-* means "between." For example, an interstate highway is a highway

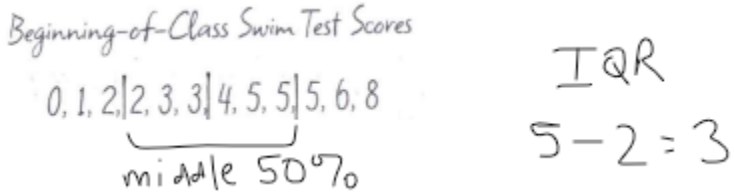
***between* two states. So, interquartile means "between quartiles." The interquartile range is the difference between the lower quartile and the upper quartile.**

Half of the values in the data set are between the lower quartile and upper quartile, so the interquartile range tells us how spread out the middle half of the data is. Have your child read the bottom section of the text box aloud.



What is the interquartile range for the end-of-class scores? *2*. **That means that the middle half of the scores are all within 2 points of each other.**

Point to part A. **What is the interquartile range for the beginning-of-class scores?** *3*. **How do you know?** *5 minus 2 equals 3*. **At the beginning of the class, the middle half of the scores were all within 3 points of each other.**



Were the scores closer together at the beginning of the class or the end of the class? *They were closer together at the end of the class*. **The interquartile range decreased, so the scores were closer together at the end of the class than at the beginning.**

Do you think the swim class instructor did a good job? Why or why not? *Sample answer: The students' scores increased overall and got closer together.*

Activity (C): Play IQR War

Play IQR War.

<u>IQR War</u>

Materials: Deck of playing cards with jacks, queens, and kings removed (40 cards total)

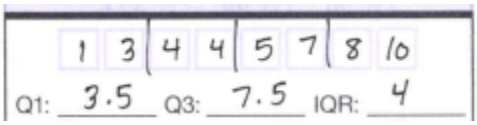
Object of the Game: Score the most total points.

How to Play: Shuffle the cards and place them in a face-down pile.

IQR War has 3 rounds. In each round, both players take as many cards from the top of the deck as are they need to complete the blanks on their score card.

- Round 1: 8 cards
- Round 2: 9 cards
- Round 3: 10 cards

Then, both players arrange their cards in order from least to greatest. They write the numbers on the cards on their score card and find Q1, Q3, and the IQR. Each player wins as many points as their IQR.



Caption: Sample round. The IQR is 4, so the player wins 4 points.

If you run out of cards, shuffle the cards in the discard pile and add them to the face-down pile.

Take turns until both players have completed 3 rounds. Each player finds the sum of their IQRs. Whoever has the greater total wins.

Advanced Version: Take one more card than blanks in each round. (For example, for the first round, take 9 cards.) Discard one card before you write your numbers on your score card.

Independent Practice and Review

Have your child complete the Lesson 15.6 Practice and Review workbook pages.

Lesson 15.7 – Box Plots

PURPOSE	MATERIALS
• Interpret and draw box plots • Use box plots to compare data sets	• Chair • Stopwatch or stopwatch app

- **What five numbers are in the five-number summary for a data set?** *The minimum, lower quartile (Q1), middle quartile (Q2), upper quartile (Q3), and maximum.*
- **What does interquartile range measure?** *The range for the middle 50% of the values in a data set.*

Warm-up: Perform the Sit-to-Stand Test

Later in this lesson, you'll draw conclusions about data from a test called the sit-to-stand test. Doctors use the sit-to-stand test to assess people's fitness, especially older adults.

To warm up, you get to try the sit-to-stand test. To perform the test, you sit in a chair and see how many times you can stand up from the chair in 30 seconds.

If your child cannot physically perform this test, skip the activity and move on to part A.

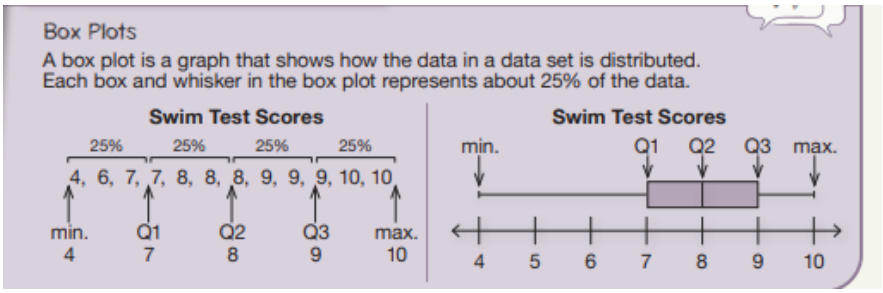
Place a straight-backed chair against a wall (to keep the chair stable). Have your child sit in the chair with her arms crossed over her chest and practice standing up and sitting back down again. Then, use a stopwatch to count many times she can stand up and sit back down again in 30 seconds.

Activity (A): Understand Box Plots

Show your child the swim test scores in part A. **In the last lesson, you learned how to find the five-number summary and interquartile range for this data set.**

What five numbers are in the five-number summary? *The minimum, lower quartile (or Q1), middle quartile (or Q2), upper quartile (or Q3), and maximum.* **The five-number summary splits the data into intervals that each contain 25% of the values. These five numbers are the endpoints of the intervals.**

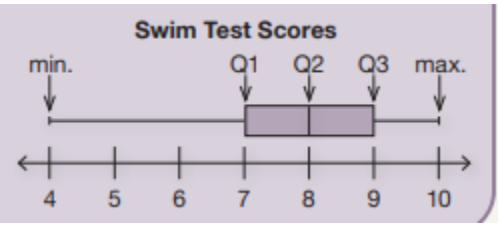
What is the five-number summary for the swim test scores? *4, 7, 8, 9, and 10.*



Today, you'll learn how to draw and interpret box plots. A box plot uses the numbers from the five-number summary to show how the data is split into intervals that each contain about 25% of the data.

Box plots are sometimes called box and whisker plots, because they have 2 boxes and 2 the lines on either side of the boxes that look like whiskers. The number line below the box plot provides the scale for the graph.

- **The left whisker goes from the minimum to Q1. The bottom 25% of the data is in this interval.**
- **The left-hand box goes from Q1 to Q2. The next 25% of the data is in this interval.**
- **The right-hand box goes from Q2 to Q3. The next 25% of the data is in this interval.**
- **The right whisker goes from Q3 to the maximum. The top 25% of the data is in this interval.**

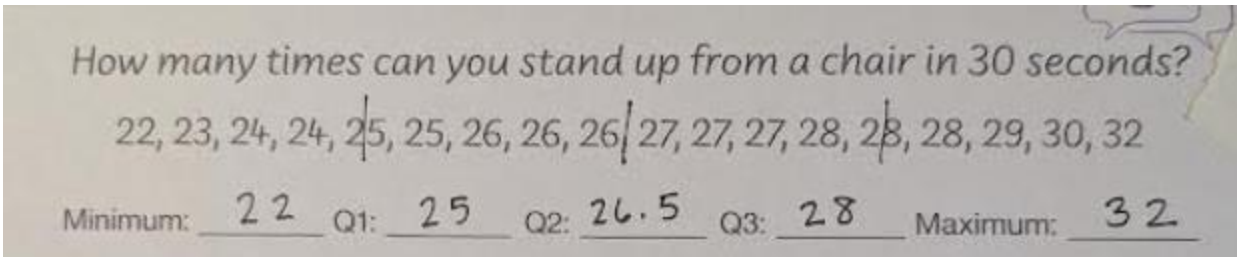


Box plots tell us a lot of information! A box plot shows the median, shape of the data, IQR, and range, all at a glance. Cover the swim test scores with your hand. Have your child use the box plot only to answer the following questions.

- **Q2 is the median, so the line between the two boxes tells us the median. What is the median of this data set?** *8*.
- **The lengths of the boxes and whiskers tells us whether the data is right-skewed, left-skewed, or symmetric. In this box plot, the left whisker is a lot longer than the right whisker. That means that the data on the left side is more spread out than the data on the right side. So, is the data set left-skewed or right-skewed?** *Left-skewed*.
- **The ends of the boxes are at Q1 and Q3, so we can use them to find the IQR. What's the IQR for this data set?** 2. **How do you know?** *9 minus 7 equals 2*.
- **The ends of the whiskers are at the minimum and maximum, so we can use them to find the range of the box plot. What's the range for this data set?** *6*. **How do you know?** *10 minus 4 equals 6*.

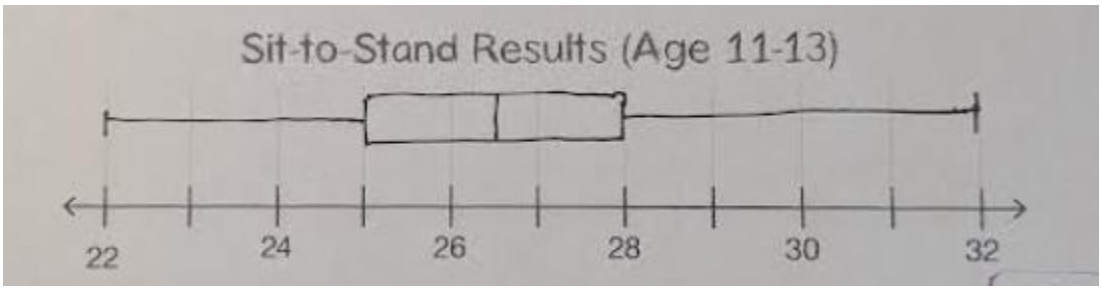
Activity (B): Draw a Box Plot

The data in part B shows how 18 children performed on the sit-to-stand test. Have your child complete the five-number summary for the data.



Now, we'll draw a box plot to match the data. It's usually easiest to draw the box first and then add the whiskers.

- **First, draw a box from Q1 to Q3.** *Child draws a box from 25 to 28.*
- **Next, draw a line through the box at Q2.** *Child draws a line through the box at 26.5.*
- **Last, draw the whiskers. Draw one whisker from Q1 to the minimum.** *Child draws a whisker from 22 to 25.* **Draw the other whisker from Q3 to the maximum.** *Child draws a whisker from 28 to 32.*

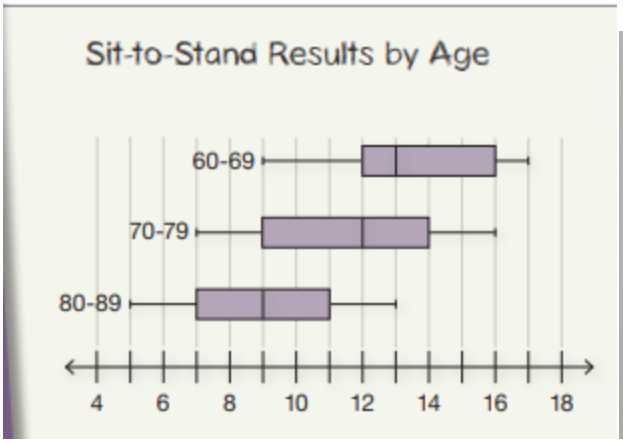


Use the following questions to discuss the box plot:

- **What is the median for the data?** *26.5.* **Half of the values are greater than 26.5 and half of the values are less than 26.5.**
- **The middle half of the values are between what two numbers?** *25 and 28.*
- **What is the IQR?** *3.* **How do you know?** *28 minus 25 equals 3.*
- **What is the range?** *10.* **How do you know?** *32 minus 22 equals 10.*
- **Is the data right-skewed, left-skewed, or roughly symmetric?** *Roughly symmetric.* **The right whisker is a little longer than the left whisker, but the right and left boxes have the same length. So, we can say that the data is roughly symmetric.**

Activity (C): Use Box Plots to Compare Data Sets

The data in part C shows how 3 different age groups performed on the sit-to-stand test. Have your child use the box plots to complete the five-number summaries in the chart.

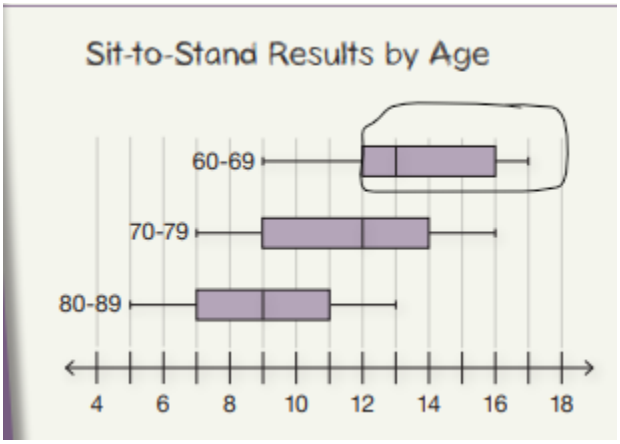


Age	60-69	70-79	80-89
Min.	9	7	5
Q1	12	9	7
Q2	13	12	9
Q3	16	14	11
Max.	17	16	13

In general, which group performed the best on the sit-to-stand test? *The 60 to 69-year-olds.* **How do you know?** *Sample answer: Their median is the highest. All the numbers in the five-number summary are higher than the other groups.*

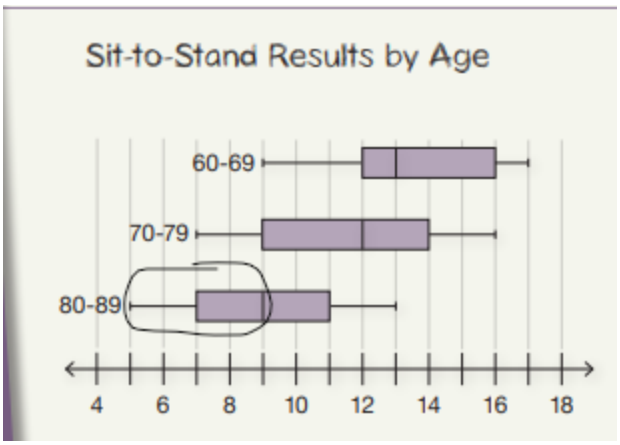
Which group performed the worst? *The 80 to 89-year-olds.* **How do you know?** *Sample answer: Their median is the lowest. All the numbers in the five-number summary are lower than the other groups.*

The median for the 70 to 79-year-olds was 12. What percentage of the 60 to 69-year-olds scored 12 or higher? *75%.* **How do you know?** *The lower quartile (Q1) for 60 to 69-year olds is 12.*



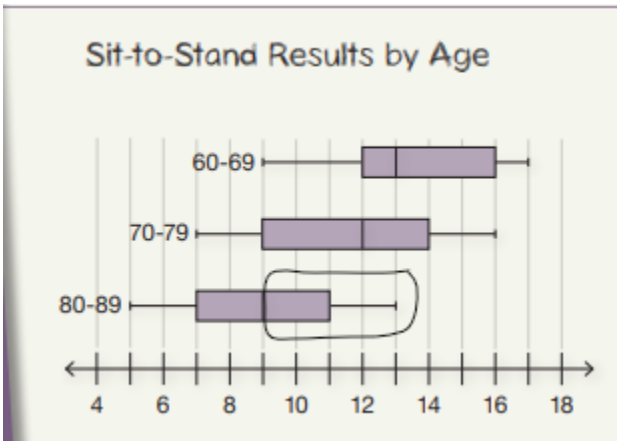
Caption: Each box and whisker represents 25% of the data. Both boxes and one whisker are greater than 12, so 75% of the data was greater than 12.

The lowest score for the 60 to 69-year-olds was 9. What percentage of 80 to 89-year-olds scored 9 or lower? *50%.* **How do you know?** *The middle quartile (Q2) for 80 to 89-year-olds is 9. Half of the values are less than Q2.*



Caption: The

Did every person in the 80 to 89 year-old-group perform worse than every person in the 60 to 69-year old group? *No.* **How do you know?** *Sample answer: The minimum for 60 to 69-year-olds was 9. 50% of the 80 to 89-year-olds scored 9 or higher.*



Independent Practice and Review

Have your child complete the Lesson 15.7 Practice and Review workbook pages.

Lesson 15.8 – Enrichment (Optional)

PURPOSE	MATERIALS
• Review memory work • Appreciate how math can be used in real-life situations • Choose a statistical question, gather data, and summarize the results • Summarize what your child has learned and assess your child's progress	• *What's the Point of Math?*

This optional enrichment lesson is in the middle of the unit so you can wrap up your study of data and statistics before moving on to the end-of-the-year review and celebration lessons.

Warm-up: Review Memory Work

Quiz your child on any of the memory work items that he struggled with during this unit.

Math Book: *What's the Point of Math?*

Read pages 90-93 and 98-101 in *What's the Point of Math?*

Enrichment Activity: Investigate a Statistical Question and Summarize the Results

In this activity, you get to choose a statistical question to investigate. We'll gather data and then use summary statistics to describe the data set's center and spread.

Statistical questions have numerical answers, and the answers aren't all the same. They vary from each other, and there isn't just one right answer. A statistical question can be answered through a survey, through measurements, or through an experiment.

- **"How many emails do you receive each day?" is a statistical question. People can answer the question with a number, and you expect that different people will give you different answers.**
- **"How much do apples weigh?" is a statistical question. When you weigh an apple, you get a number. You expect that different apples will weigh different amounts.**
- **"When you roll two dice, what is their sum?" is also a statistical question, because you can roll two dice multiple times and record their sum. You don't expect to always get the same sum.**

A question like "What is your favorite season?" is not a statistical question, because it doesn't have a numerical answer. A question like "How tall is the Eiffel Tower?" is not a statistical question, because there is only one answer.

What statistical question would you like to investigate? *Answers will vary.* Discuss possibilities with your child and help him choose one. If he has trouble, encourage to choose one of the example questions from the discussion above.

Then, help your child decide how to gather his data. He should aim to collect 10-15 data points.

Have your child find the minimum, maximum, range, mean, median, and mode for his data set. Then, have him find the quartiles and interquartile range and make a box plot to match.

Unit Wrap-Up

Have your child complete the Unit 15 Wrap-Up.